1A) If $n(P)=1$, this means there is an even number of negations in P. Illustration:

I) $n(\neg\neg\neg a \vee \neg b) = n(\neg\neg\neg a)\cdot n(\neg b) = -1\cdot n(\neg\neg a)\cdot -1\cdot n(b) =$
$-1\cdot -1\cdot n(\neg a)\cdot -1\cdot 1 = -1\cdot -1\cdot n(a) = 1$

II) $n(a \vee \neg(b \vee c)) = n(a)\cdot n(\neg(b\vee c)) = 1\cdot -1\cdot n(b\vee c) = -1\cdot n(b)\cdot n(c) = -1$

This is as expected, since wff I has an even number of negations in it and wff II has an odd number.

B) BASIS Take an arbitrary propositional parameter a. a is a wff in both $L_{\rightarrow\wedge}$ and $L_{\neg\vee}$. $v(a)=v(a)$ for all valuations.

IH Take two arbitrary wffs in $L_{\rightarrow\wedge}$, A and B, and suppose that there are some wffs in $L_{\neg\vee}$, C and D, such that $v(A)=v(C)$ for all valuations and $v(B)=v(D)$ for all valuations.

IS Case $\rightarrow$: $v(A\rightarrow B)=v(\neg A\vee B)$ for all valuations, and $\neg A\vee B$ is a wff in $L_{\neg\vee}$.

Case $\wedge$: $v(A\wedge B)=v(\neg(\neg A\vee\neg B))$ for all valuations, and $\neg(\neg A\vee\neg B)$ is a wff in $L_{\neg\vee}$.

A	B	$A\rightarrow B$	$\neg A\vee B$	$A\wedge B$	$\neg(\neg A\vee\neg B)$	
1	1	1	1	1	1	0
1	0	0	0	0	0	1
0	1	1	1	0	0	1
0	0	1	1	0	0	1

CONCLUSION Therefore, for each wff A of $L_{\rightarrow\wedge}$, there is some wff B of $L_{\neg\vee}$ such that, for all valuations, $v(A)=v(B)$.

3A) $((p \wedge q) \supset r) \wedge ((q \wedge r) \supset s), +$

$(p \wedge q) \supset s, -$

$(p \wedge q) \supset r, +$

$(q \wedge r) \supset s, +$

$\neg(p \wedge q), -$ $*$

$s, -$

$\neg p, -$

$\neg q, -$

CAN ALREADY CLOSE HERE

$\neg(p \wedge q), +$ $***$ | $r, +$

$\neg p, +$ X | $\neg q, +$ X | $\neg(q \wedge r), +$ $**$ | $s, +$ X

$\neg q, +$ X | $\neg r, +$

! erratum

$*$: missing "$\neg p \vee \neg q, -$"

$**$: missing "$\neg q \vee \neg r, -$"

$***$: missing "$\neg p \vee \neg q, +$"

There is a complete and open branch. Hence, the inference is invalid and we can specify a countermodel:

$p \rho 1$

$q \rho 1$

$r \rho 0, r \rho 1$

$s \rho 0$

Nothing else obtains about p, q, r, and s.

B) The inference is valid in K_3. All branches in the above tableau close, because it is not possible for a formula in K_3 to be both true $(r, +)$ and not true $(\neg r, +)$.

5)

$\Diamond p, 0$

$\neg((\Box\neg p \supset \Box(p \supset q)) \wedge (\Box p \supset \Diamond(q \supset p))), 0$

$0r1$

$p, 1$

$\neg(\Box\neg p \supset \Box(p \supset q)), 0$	$\neg(\Box p \supset \Diamond(q \supset p)), 0$
$\Box\neg p, 0$	$\Box p, 0$
$\neg\Box(p \supset q), 0$	$\neg\Diamond(q \supset p), 0$
$0r2$	$\neg(q \supset p), 1$
$\neg(p \supset q), 2$	$q, 1$
$p, 2$	$\neg p, 1$
$\neg q, 2$	$\times$
$\neg p, 2$	
$\times$	

All branches close. Therefore, the inference is valid.

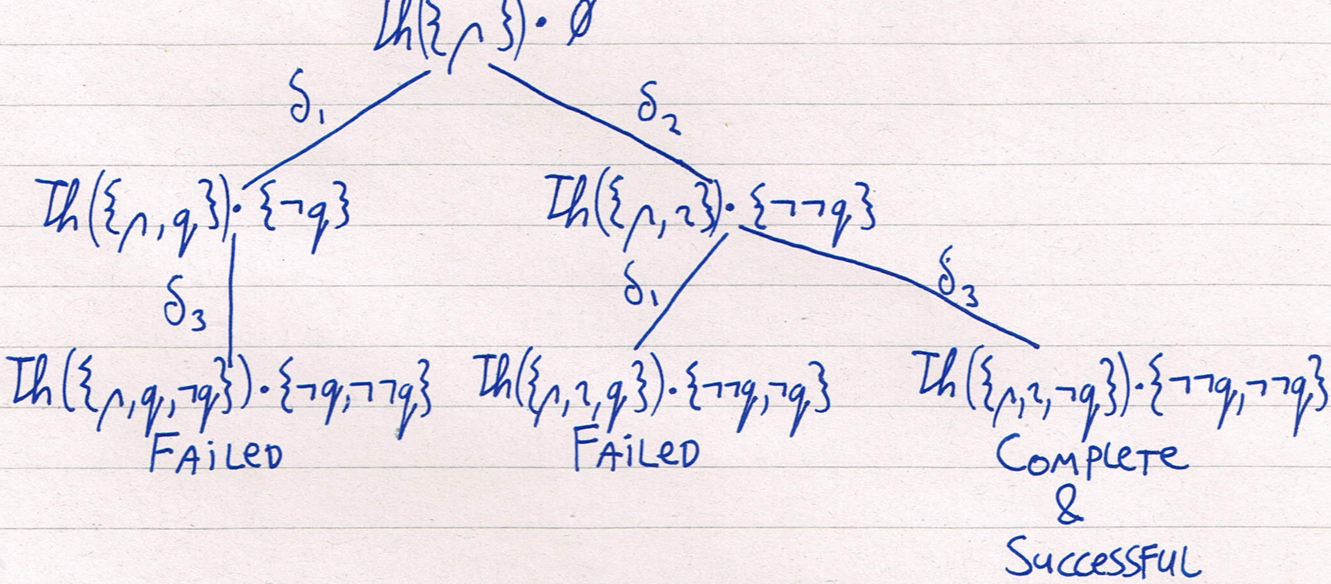
7A) Yes, as shown by the function f defined as follows:

$f(0) = w_2$, $f(1) = w_0$, $f(2) = w_1$

B) No. In the induced interpretation:

- W is infinitely large
- R is extendable, meaning every world is related to at least (and in this case exactly!) one other world.
- the values of $v(A)$ and $v(B)$ are arbitrary for all worlds except w_1.

9A)



B) Yes. There is only one extension of this theory, Th({p, r, ¬q}), and it contains p.

C) No. The aforementioned extension does not contain q.

D) When it has no extensions.